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The mechanics of the formation region of vortices behind bluff bodies

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The characteristic lengths of the oscillating wakes of bluff bodies is discussed; in particular, those used in the universal non-dimensional frequencies proposed by Roshko (1954*b*) and Goldburg, Washburn & Florsheim (1965). It is concluded that these are equivalent at high Reynolds number. A closer examination leads to the conclusion that there are two simultaneous characteristic lengths; the scale of the formation region, and the width to which the free shear layers diffuse. Discussion of the mechanics of the formation region results in a physical basis for the determination of the frequency by these two characteristic lengths. The ideas developed are applied to the effects of splitter plates in the wake. The possibility of a high-Reynolds-number symmetrical formation region is suggested as an explanation of the very small lift values observed in the absence of free-stream disturbances.

1. Wake width as characteristic length

In a recent publication Goldburg *et al.* (1965) show that the 'total wake momentum thickness,' θ, is the relevant characteristic length to be used in the formation of a universal non-dimensional frequency, $S_\theta = N\theta/U$, where N is the fundamental wake frequency and U is the free-stream speed. They make no reference to the work of Roshko (1954*b*), in which he defines a similar universal number $S^* = Nd'/U_s$ where d' is the wake width determined by his (1954*a*) notched-hodograph theory, and U_s is a wake velocity simply related to the base pressure coefficient C_p. The coefficient of drag per unit length, C_D, is similarly related to C_p by the notched hodograph theory. Goldburg *et al.* show that for two-dimensional flow $\theta/d = \frac{1}{2}C_D$, where d is the length used in forming the drag coefficient.

Roshko achieves good collapsing of the frequency as a function of Reynolds-number curves for the circular cylinder, normal flat plate and 90° wedge at the higher Reynolds numbers. There is some separation of the curves for two of the bodies at a reduced Reynolds number, R^*, of 4000. When, however, results for a circular cylinder from Gerrard (1965) are compared with Roshko's S^*, R^* plot, considerable discrepancy is found. This is because the Strouhal number (Nd/U) is the same in both experiments, but the values of d' and U_s are quite different, in the low free-stream turbulence measurements, from the values Roshko

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determines. At $R^* = 4000$, Roshko's values of S^* lie between 0·16 and 0·20, Gerrard's value of S^* at this reduced Reynolds number is 0·29.

The results of Roshko and Gerrard have also been examined on the basis of a plot of S_θ as a function of R_θ following Goldburg *et al.* There is again considerable spread of the results at values of R_θ of the order of 1000. The spread is much less at higher Reynolds numbers though not as small as the result Roshko publishes.

There seems little to choose between the basic ideas of Roshko and of Goldburg *et al.* as the Reynolds number approaches the critical. The ratio of the non-dimensional frequencies

$$\frac{d'}{U_s}\bigg/\frac{\theta}{U} = \left(\frac{d'}{d}\frac{U}{U_s}\right)\frac{2}{C_D}.$$

When this is computed for a circular cylinder it is found to be a quite rapidly varying function of Reynolds number below a Reynolds number of 30,000. Values of C_D were taken from Goldstein (1938), U_s/U from Gerrard (1965) and d'/d from Roshko (1954*b*). At a Reynolds number of 3000 the ratio is 1·9 or greater. The ratio steadily decreases with increasing Reynolds number to about 1·3 at $R = 30{,}000$. Reasonable extrapolation suggests that the ratio does not fall below 1·2 by the time the critical Reynolds number is reached. It appears that Roshko's suggestion is correct, that a single parameter (R_* or R_θ) insufficiently describes the phenomenon when the position of transition is moving towards the body in the shear layers which later roll up into vortices. Unfortunately, this transition region extends from a Reynolds number of about 400 to one of about 40,000. Examination of Roshko's (1954*b*) figure 5 strongly suggests that when the flow is entirely laminar the Strouhal number itself is the required universal non-dimensional frequency. At high Reynolds number, Roshko's and Goldburg's methods are equally good in producing a universal non-dimensional frequency. Both produce discrepancies as the relative proportion of laminar shear layer in the formation region increases.

2. Two simultaneous characteristic lengths

Many of the flow parameters are dependent on the turbulence level of the free stream: as the author (1965) has shown, this is reflected in a dependence on cylinder diameter as well as on the Reynolds number. There are only very small concomitant changes in the Strouhal number. Figure 1, obtained at low turbulence level, shows that the average Strouhal number is no different from that obtained at higher turbulence level and that the exact form of the relationship varies from day to day, presumably with the disturbance level in the wind tunnel. The constancy of the Strouhal number is as yet unexplained. On Roshko's theory, referring to the values of U_s determined by the author (1965), we would expect just as large a change in the Strouhal number as we observe in U_s: this change would be amplified by the attendant change in d'.

Roshko (1954*b*) considers average values of d' and U_s, which in his experiments varied little over the whole Reynolds-number range. It is instructive to apply Roshko's method in detail: the experimental results of Roshko (1954*b*) and Gerrard (1965) are plotted in figure 2. Clearly S_* is not a universal non-dimensional frequency.

When there is no interference, in the form of splitter plates, in the wake, one may expect that the width d' and the length l_f of the formation region will be equally good characteristic lengths. When splitter plates are inserted in the wake we shall see below that l_f appears, in some cases at least, to be more significant. The use of an experimental quantity rather than d' is also an attraction.

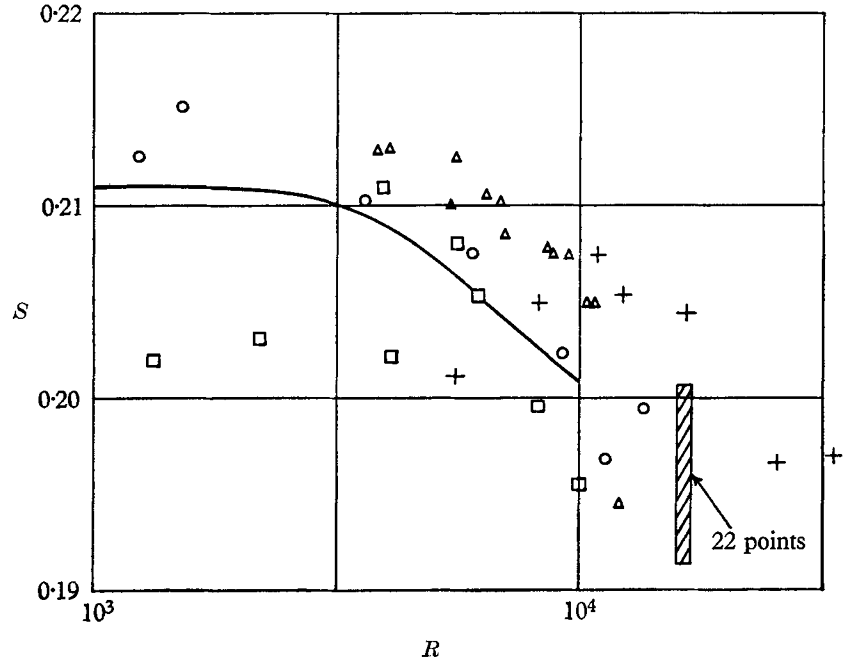

FIGURE 1. Strouhal number as a function of Reynolds number in a low-turbulence free stream. Symbols represent measurements in the same run. Open symbols, cylinder diameter $\frac{1}{4}$ in. : + and the 22 points collected together, cylinder diameter 1 in. The curve is reproduced from Roshko (1954*b*).

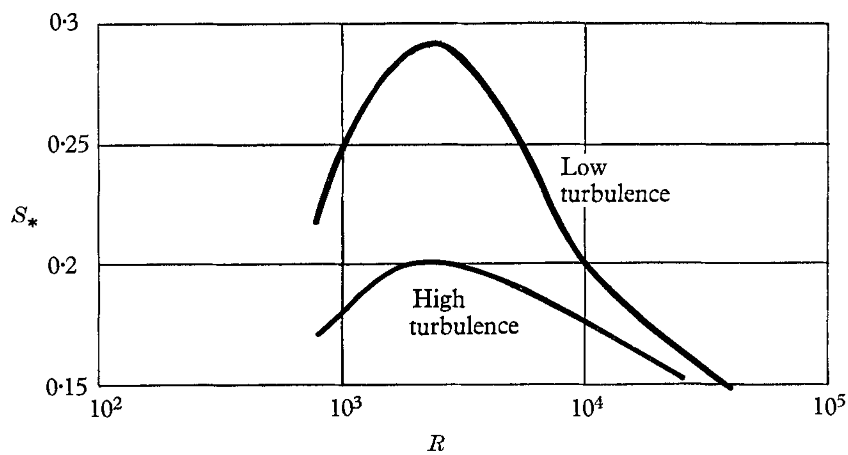

FIGURE 2. Roshko's (1954) non-dimensional frequency $S_* = Nd'/U_s$ as a function of Reynolds number. The high-turbulence curve reproduced from Roshko (1954*b*), the low-turbulence curve from Gerrard (1963).

We shall consider, therefore, a non-dimensional frequency Nl_f/U or Sl_f/d. Little qualitative difference results from the use of U rather than U_s. The velocity chosen should be representative of the shear layers before they roll up into vortices. Such a speed is expected to lie between U and U_s. The non-dimensional

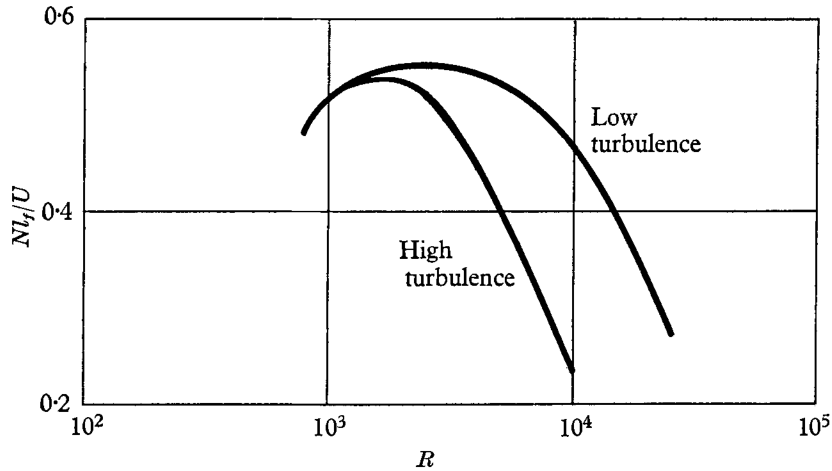

FIGURE 3. The first characteristic length, l_f the length of the formation region. $S' = Nl_f/U$ is plotted as a function of Reynolds number. It is assumed that the observed (Bloor 1964) dependence on cylinder diameter reflects a dependence on free stream turbulence.

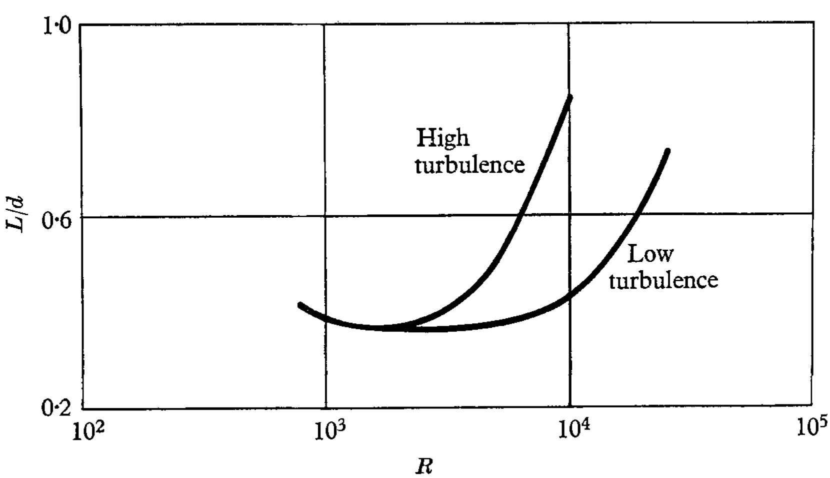

FIGURE 4. The second characteristic length, L, the diffusion length, plotted as a function of Reynolds number. L is defined such that $S'L/d = 0{\cdot}2$.

frequency $S' = Nl_f/U$ is plotted in figure 3. The assumption is made that Bloor's (1964) small cylinder-diameter values of l_f apply to the higher free-stream turbulence case.

We shall see in § 6 that this assumption may be incorrect and that, as with d', we expect the maximum l_f in the turbulent stream to be significantly less than the maximum l_f in the undisturbed flow. This has no qualitative effect on the arguments proposed before § 6.

We now introduce a second characteristic length L, such that $S'L/d$ is independent of Reynolds number. The values of L/d corresponding to $S'L/d = 0{\cdot}2$ are plotted in figure 4. The value 0·2 is arbitrarily chosen, but the resulting value of L agrees in order of magnitude with the physical interpretation which will be suggested below.

After a physical discussion of the mechanics of the fluid flow in the formation region, we shall return to the problem of seeking opposing tendencies which result in the constancy of the Strouhal number. Insufficient data are available to take the discussion beyond a consideration of the circular cylinder.

3. The size of the formation region

The author (1965) has shown the length of the formation region of the vortices behind a circular cylinder to be a relevant length scale for the distribution of fluctuating velocity with distance close to the body. The variation of length of the formation region and of Roshko's (1954*b*) wake width d', with Reynolds number, is similar when the free-stream turbulence level is low.

Little has been published on the region of flow close behind a bluff body, which it is recognized, plays an important role in the determination of the frequency of vortex shedding. A physical discussion of the mechanics of the

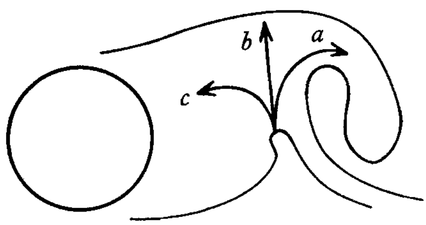

FIGURE 5. Filament-line sketch of the formation region. Arrows showing reverse flow (c) and entrainment (a) and (b).

formation region will serve to introduce a relationship between the size of the formation region, the strength of the vortices and the frequency of vortex shedding. The most dramatic changes take place in the formation region as the Reynolds number is increased from about 10^3 to about 5×10^4 and the transition to turbulence moves upstream in the shear layers (Schiller & Linke 1933, and Bloor 1964). During this movement, the formation region shrinks in size for a reason which we can make clear by reference to figure 5.

Bloor (1964) defines the end of the formation region as the point on the wake axis, closest to the cylinder at which a hot-wire signal, characteristic of the oscillating wake downstream, is observed. This implies that the end of the formation region is where fluid from outside the wake first crosses the axis. Fluid is drawn across the wake by the action of the growing vortex on the other side. Figure 5 illustrates the formation region by means of filament lines within the rolling-up shear layers. It is shown at the instant when irrotational flow is beginning to cross the wake axis. The arrows show the path of this fluid at later times: it is partly entrained by the growing vortex and partly by the shear layer upstream of the vortex. Some of this fluid will also find its way into the interior of the formation region. The fluid must bear vorticity of opposite sign to that of the entraining layer because the irrotational flow entering the wake cannot cross the filament line springing from the lower boundary-layer separation point in figure 5.

The entrainment of fluid bearing vorticity from the other shear layer takes place periodically, but entrainment by the shear layer is a continuous process. The vorticity in the interior of the formation region, apart from the periodic flow described above, may be expected to be considerably less than that of the shear layers. In this case, the periodic entrainment of fluid across the wake will have a predominant influence.

The size of the formation region is determined by the balance between entrainment into the shear layer and the replenishing of fluid by the induced reversed flow described above. Changes in entrainment with Reynolds number will be governed by the way in which the flows, marked a, b and c in figure 5, separately vary. We can argue *a posteriori* that the circulation carried by flow a does not vary significantly with Reynolds number. The entrainment flow b will be governed mainly by the length of the turbulent shear layer. As the circulation carried by flow b increases with Reynolds number, we must assume that that carried by flow c decreases. Much of the circulation of flow c will remain in the interior to be effectively cancelled half a period later. Some cancellation will occur at the rear surface of the body.

If from the equilibrium situation, at one Reynolds number, the Reynolds number is increased, the position of transition will move towards the cylinder and the rate of entrainment will tend to increase. The reversed flow is unable to increase because the forming vortex will tend to be weaker the greater the entrainment, since fluid bearing oppositely signed vorticity will, on the whole, be entrained. Hence the formation region must shrink in length.

The validity of this argument is substantiated by the following simple treatment. By integrating the turbulent shear-layer velocity distribution given by Schlichting (1955), we find that the volume flux increases like the mean speed multiplied by the distance from the origin of the layer (at large distances from the origin). If we neglect the laminar entrainment compared with that of the turbulent part of the layer, we may put the rate of entrainment proportional to $U_s l_t$, where U_s is the speed outside the boundary layer at separation, and l_t is the length of turbulent shear layer in the formation region. This volume flux must balance the reversed flow if the formation region is to remain the same size each period. At low Reynolds numbers, where the vortices are laminar, let us make the assumption that the entrainment flow b is negligible compared with that under turbulent conditions found at higher Reynolds number. If the entrainment bears fluid of opposite vorticity to that in the shear layer, we expect the reduction in the strength of the vortices below the laminar value to be proportional to the difference in turbulent entrainment. That the strength of laminar vortices is considerably less than the circulation shed in one period from the separation point, implies that the circulation carried by flow a is quite large. We have assumed that the flow a circulation (suitably non-dimensionalized) does not vary with the Reynolds number.

Using vortex strengths determined by Bloor & Gerrard (to be published), and the lengths of turbulent shear layer determined by Bloor (1964) we find that the proportionality does approximately exist between the rate of turbulent entrainment and the change in vortex strength. This is shown in figure 6. The differences in vortex strength are plotted non-dimensionally as the strength of laminar vortices, taken to be 0·8, minus the strength of the turbulent vortices at the particular Reynolds number. The vortex strength K is non-dimensionalized by division by πUd. The bifurcation of the entrainment curve

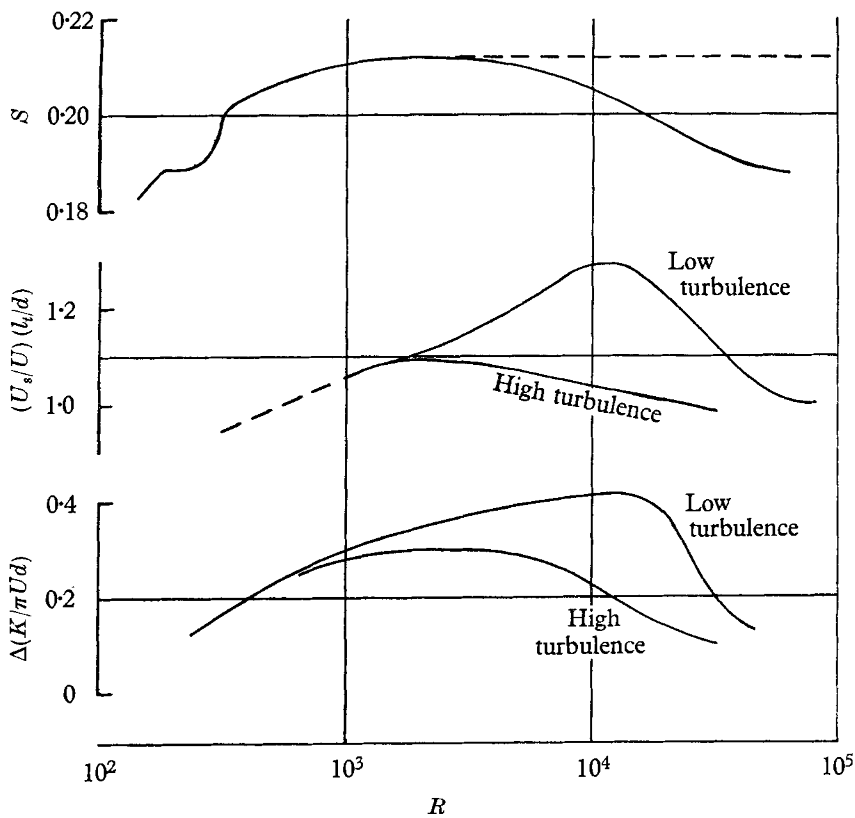

FIGURE 6. Strouhal number $S = Nd/U$, entrainment volume flow $(U_s/U)\,(l_t/d)$, and laminar vortex strength minus turbulent vortex strength, as a function of Reynolds number. The high-turbulence curves are derived from Roshko (1954*b*) the low-turbulence curves from Gerrard (1965).

corresponds to the dependence upon diameter as well as Reynolds number referred to above. Curves corresponding to high and low turbulence level are also shown in the vortex strengths. The high-turbulence vortex strengths are determined from the results of Roshko (1954 *b*), as will be explained in the publication by Bloor & Gerrard. Thus it is shown that the changes in the strength of the vortices with Reynolds number can be attributed to the variation of entrainment of vorticity-bearing fluid by the turbulent part of the shear layer.

4. The frequency of vortex shedding

The growing vortex continues to be fed by circulation from the shear layer until the vortex becomes strong enough to draw the other shear layer across the wake. The approach of oppositely-signed vorticity in sufficient concentration cuts off further supply of circulation to the vortex, which then ceases to increase in strength. We may speak of the vortex as being shed from the body at this stage. We postulate that this is the basic mechanism determining the frequency of vortex shedding. Roshko (1954*b*) has shown that the frequency increases if the scale of the formation region is reduced. When the shear layers are brought closer together their interaction is facilitated and the periodic time shortened.

We have already shown that, though the above may be described as the basic mechanism, there is another major effect. This we suggest to be the thickness of the shear layer when it reaches the region of strong interaction at the end of the formation region. The significance of this was implied in Fage & Johansen's paper in 1927, where the mean thickness of the shear layers springing from different bodies was shown. The importance of the shear-layer thickness was explicitly mentioned by Berger in 1964. We do not differentiate between a thick shear layer spread under the action of turbulent diffusion and an effectively thick shear layer formed by its rolling up into a succession of concentrated vortices. An increase in the turbulence in the shear layers will result in their being more diffuse in the region of interaction. When the layer is diffused it will take longer for a sufficient concentration of vorticity to be carried across the wake and initiate shedding. So we expect the shedding frequency to decrease as the 'diffusion length' L increases. On the same reasoning, the greater diffusion of the vorticity will result in less entrainment into the growing vortex, hence a higher value of the vortex strength at high Reynolds number. In this case there will be an effective cancellation of circulation in the interior of the formation region.

The relative constancy of the Strouhal number over the whole Reynolds-number range from about 400 to the critical Reynolds number follows from the fact that the two major frequency-determining factors tend to change the frequency in opposite directions as the Reynolds number is altered (compare figures 3 and 4). When the free-stream turbulence level is increased from a low value at constant Reynolds number, the length of the formation region decreases, but the diffusion length presumably increases and to such an extent that the opposing frequency changes balance. That they balance is not fortuitous for the increased entrainment attending the increase in turbulence level is responsible for both the decrease in size of the formation region and for the increase in the diffusion length. The width of a turbulent shear layer and the entrainment into it are both proportional to the length of the layer.

Two major factors which determine the frequency of vortex shedding have been postulated. Kronauer (1964) has proposed the theory that the frequency is mainly determined by the feedback of velocity fluctuations to the boundary-layer separation point from the wake. It is the author's contention that, while this feedback is essential to the production of the oscillating wake, it is not the primary factor in the determination of the frequency. Work by the author, as yet unpublished except in outline (Gerrard 1963), suggests that without this feedback, which causes the separation points to oscillate, the shear layers will roll up without significant interaction: the shear layer will escape entrainment and the frequency-determining mechanism will have no opportunity to work. Independent roll-up of the shear layers is characteristic of the observed wake development between Reynolds numbers of about 40 and 90.

At higher Reynolds numbers, the turbulent character of the frequency-determining mechanism explains why the frequency is not well defined. The spectra of oscillating quantities are narrow bands of noise rather than pure tones.

5. The effect of splitter plates in the wake

To be acceptable, any theory of the frequency-determining mechanism must be in accord with the observed effects of splitter plates in the wake of bluff bodies. In § 2 the constancy of the Strouhal number over a wide range of Reynolds number is attributed to the constancy of the product $l_f L$. Thus we expect the Strouhal number to increase if l_f or L decrease.

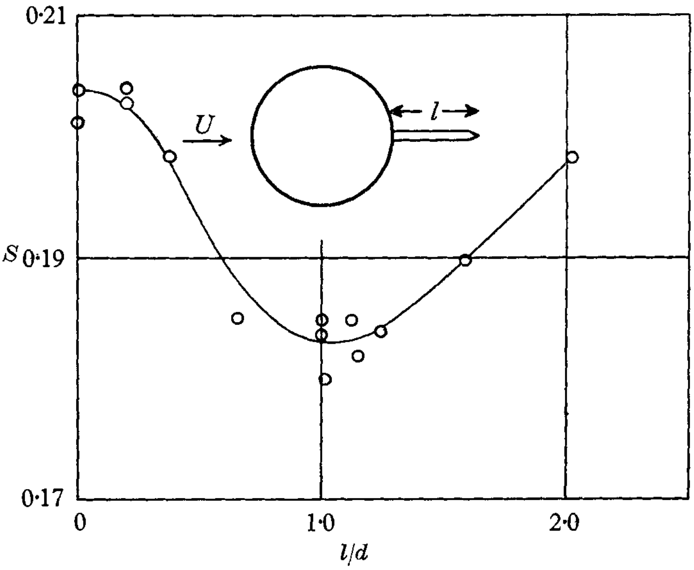

FIGURE 7. Strouhal number as a function of plate length with a splitter plate on the wake axis.

Some experiments were made at a Reynolds number of 2×10^4 on a 1 in. diameter cylinder with a splitter plate attached as shown in figure 7. The frequency of vortex shedding was determined with a hot-wire anemometer in the wake and a wave-analyzer attached to the amplifier output. At this Reynolds number the length of the formation region was expected to be about one diameter (that is, to terminate one radius downstream of the rear of the body) since the Pennsylvania State University low-speed wind tunnel did not have a low-turbulence level.

Figure 7 shows that as the length of the splitter plate is increased up to and even beyond the length of the formation region, the Strouhal number is decreased. This decrease is expected if the length of the formation region is increased by the insertion of the plate. The diffusion length will also increase if l_f increases. Physically, we expect the cross-flow resulting in the shedding of the eddy to be less easily set up in the presence of the splitter plate, some of the cross-flow will be deflected away from the other side of the wake; this results in an increased period. The production of circulation at the rear of the cylinder is expected to increase when a splitter plate is present. This results in the weakening of the vortex strength in the early stages of its growth which would also result in a decreased frequency. This effect is expected to be small.

This experiment shows the fundamental importance of the flow in the interior of the formation region in determining the frequency.

Experiments were also made repeating the work of Roshko (1954*b*) using splitter plates with a gap between the cylinder and the plate. The frequency measurements were made at a cylinder Reynolds number of 2×10^4 again, which was close to that used by Roshko. When the gap length was small, the reduction in Strouhal number (for the smaller plate lengths) was the same as that found with no gap. For larger gaps there is a large effect of the gap or the leading edge of the plate. It was found that the relevant parameter was the width of the gap rather than the position of the trailing edge of the plate.

Flow visualization showed that the spectacular discontinuity in the Strouhal number observed by Roshko (1954*b*) was a transition between a flow régime in which the formation region was lengthened so as to include the plate in its interior, and a flow régime in which vortices formed upstream of the plate. The discontinuities were found to occur at the same gap length; the position of the trailing edge of the plate had only a minor effect for plate lengths of 0·7 to 1·14 diameter. The greater reduction of frequency produced by the Roshko-type plate with a gap may be explained: if some vorticity crosses to the other side of the wake upstream of the plate in insufficient quantity to induce shedding, it will be longer before the weakened vortex will eventually be shed downstream of the plate. As soon as the cross-flow upstream of the plate is capable of producing shedding, the flow will undergo transition to the higher-frequency mode.

A series of measurements of frequency were made with one plate 0·69 diameters in length placed normal to the free stream behind the circular cylinder at the same Reynolds number. Again there was a large discontinuity in Strouhal number as the distance of the plate from the cylinder was varied; as shown in figure 8. Though the jump is referred to as a discontinuity in both this experiment and the one described above, there was a range of position close to the jump in which the flow intermittently displayed both frequencies. Again the formation region expanded to include the plate when it was close to the body. At first sight this appears to contradict our expectation that a larger formation region is expected to generate a lower frequency. As far as the shedding of the growing vortex is concerned, however, the effective formation-region length is the distance from the plate to the end of the region. We conclude that the reduction in this length is responsible for the increase in Strouhal number. A physical explanation of the mechanism is also possible. When a vortex is growing close behind the plate, there is a large cross-flow velocity produced near the plate; this will facilitate the shedding process and increase the frequency. The channelling of the flow between the plate and the vortex will also reduce the effective diffusion length L of the layer. Reference must be made to the experiments of Bearman (1965), in which a splitter plate parallel to the free-stream direction produced an increase in Strouhal number. In his work, the boundary layers were turbulent on separation and the total boundary-layer thickness at the trail-

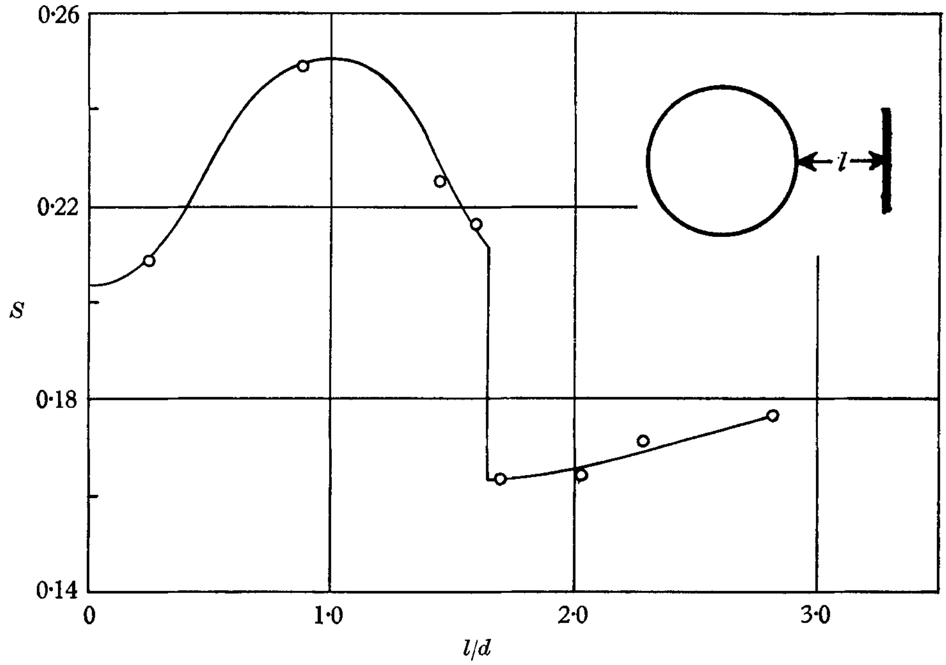

FIGURE 8. Strouhal number as a function of plate-cylinder separation with a splitter plate normal to the wake axis.

ing edge of the body was one half of the base height. It appears that entrainment by the separated layers reduces the wake width by a tendency toward reattachment of the boundary layer downstream of the step. The length of the formation region is reported to increase, however. This experimental result only fits in with Roshko's (1954*b*) finding that the more bluff the body, the lower the frequency. It seems likely that this different configuration, in which the shear layers are not considerably thinner than their separation, has different properties from the wide wakes we have considered above.

6. The effect of free-stream turbulence on vortex formation

An explanation has been offered of the relative constancy of the Strouhal number over the whole range of Reynolds number in which turbulent vortices are observed. Similarly, on the basis of the two characteristic lengths determining the frequency, a physical explanation of the effects of splitter plates has been given. It appears that one may explain the effect of increasing the free-stream turbulence from very small values in the same way, as we did in § 2. On increasing the turbulence level, the scale of the formation region shrinks and one may hypothesize that the diffusion length will increase. Opposing tendencies of frequency variation could cancel out, and hence the Strouhal number be little affected. This explanation has an unsatisfying facet.

The author (1965) has shown that when the disturbance level is very low, the fluctuating lift and fluctuating velocity at the shoulder of the cylinder change by about two orders of magnitude between Reynolds numbers of 2000 and 10^5. The variation of the vortex strength and the Strouhal number is less than 50 %. It seems unlikely that these facts can be explained simply by a change in scale of the formation region. A change in length of the formation region by a factor of two would be expected to change the amplitude of the lift by less than a factor of four. Further, a change in the free-stream turbulence does not alter the range of variation of the length of the formation region but only the Reynolds number at which it occurs. This statement depends upon our assumption that Bloor's (1964) values of l_f, obtained with small diameter cylinders, correspond to the high-turbulence case. Roshko's (1954) wake width d', on the other hand, attains higher values in the range, $700 < R < 7000$, when the disturbance level is low, than its maximum value at higher-turbulence levels. The lift also only drops to low values when disturbances are absent. Thus we are lead to suggest a possible fundamental change in the mode of vortex formation. There is a possibility that, as suggested in § 4, in the absence of free-stream disturbances, the vortex sheets are able to develop independently of each other. This implies a transition to a type of flow similar to, if not the same as, that obtained at very low Reynolds numbers. The shear layers in this case will be turbulent as Bloor (1964) has shown. Though it must be pointed out that it has not been shown that lift or velocity fluctuation at the shoulder of the cylinder fall to low values when there is a probe in the shear layer downstream.

Laminar vortices, it is believed, are usually formed in the manner described in § 4 down to a Reynolds number of about 90; and so a reversion simply to laminar shear layers would not produce an explanation of the observed fall in lift which seems to require a symmetrical configuration, whether laminar or turbulent. The possibility of a symmetrical configuration, with two standing vortices, is present only at a Reynolds number of a few thousand, because it is only in this range of Reynolds number that the formation region is large. In this condition, the removal of disturbances will allow independent development of the two shear layers. If the explanation of the extremely low lift forces observed is to be found in the idea of a symmetrical formation region, another difficulty appears. It has been considered that the wake oscillation observed at low Reynolds number with symmetrical flow near the body is due to the instability of the narrow wake. If at high Reynolds numbers the same is true, why is the Strouhal number the same with symmetrical and asymmetrical formation regions? Why, indeed, is the Strouhal number only just detectably different at the low Reynolds-number transition? One also wonders whether three-dimensional effects would need to be absent to allow the formation of the symmetrical pattern.

7. Conclusion

We conclude that the entrainment of fluid from the interior of the formation region and its replenishment by reversed flow is fundamental to the determination of a scale which determines the frequency of vortex shedding. A second characteristic length of fundamental importance to the determination of vortex-shedding frequency is, what we have termed, the diffusion length. This is the thickness of the shear layer at the end of the formation region where the layer is drawn across the wake. The cross-flow presents fluid for entrainment which bears vorticity of opposite sign to that of the shear layer which is entraining the major portion of the fluid at that instant. The physical basis for this choice of two simultaneous characteristic lengths of the shedding process is discussed.

On the basis of this mechanism the effects of splitter plates on the frequency of vortex shedding have been explained in the case of bluff bodies with wide wakes.

It is suggested that there is a possible high Reynolds-number flow régime in which the formation region is symmetrical in the absence of free-stream disturbances. This could account for the diminutive lift values observed under these conditions (Gerrard 1965).

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